Isotope analytical uncertainty propagation

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Here I (1) state the uncertainty propagation rules plainly, (2) apply them to our mass-balance model equation and calculate the results, and finally (3) I explore the derivation of Genereux’s (1998) isotopic tracer component uncertainty equations and how they, sadly, differ to my arithmetic propagation in section (2).

1 Uncertainty propagation rules

Our uncertainty rules are as follows. Let \( q \) be some quantity, and \( u_q \) be its uncertainty, where \( q \) is a function of variables \( x \), \( y \) and \( z \).

For **constants**

\[
q = Bx
\]
\[
\frac{u_q}{q} = \frac{u_x}{x}
\]

For **sums**

\[
q = x \pm y \pm z
\]
\[
u_q = \sqrt{u_x^2 + u_y^2 + u_z^2}
\]

And for **products/quotients**

\[
q = xyz
\]
\[
\frac{u_q}{q} = \sqrt{\left(\frac{u_x}{x}\right)^2 + \left(\frac{u_y}{y}\right)^2 + \left(\frac{u_z}{z}\right)^2}
\]
2 Genereux’s (1998) uncertainty propagation

Let \( p_E \) be the proportion of streamflow derived from rainfall according to an isotope \( E \), where

\[
p_E = \frac{E_{\text{streamflow}} - E_{\text{baseflow}}}{E_{\text{rain}} - E_{\text{baseflow}}}
\]  

(2.1)

We then average \( p_{\delta^2 H} \) and \( p_{\delta^{18} O} \), each with their own uncertainty derived above, as follows

\[
p = \frac{p_{\delta^{18} O} + p_{\delta^2 H}}{2}
\]  

(2.2)

\[\therefore u_p = \frac{\left| p \right| \sqrt{u_{p_{\delta^{18} O}} + u_{p_{\delta^2 H}}}}{p_{\delta^{18} O} + p_{\delta^2 H}}\]

For our study, we combined long term analytical precision and accuracy using Equation 1.2

\[
u_{\delta^{18} O} = \sqrt{0.0049 + 0.0169} = 0.1476482
\]

\[
u_{\delta^2 H} = \sqrt{0.04 + 2.25} = 1.5132746
\]

Knowing Equations 1.1–1.3, applying them to Equation 2.1 Genereux derived the following (see Equation 4 in Genereux 1998)

\[
u_{p_E} = \sqrt{\left(u_{E_{\text{baseflow}}} \left(\frac{E_{\text{rain}} - E_{\text{streamflow}}}{E_{\text{rain}} - E_{\text{baseflow}}}\right)^2 + \left(u_{E_{\text{rain}}} \frac{E_{\text{streamflow}} - E_{\text{baseflow}}}{E_{\text{rain}} - E_{\text{baseflow}}}\right)^2 + \left(u_{E_{\text{streamflow}}} \frac{-1}{E_{\text{rain}} - E_{\text{baseflow}}}\right)^2\right)}
\]

Since we know that, for our analyses, we have identical analytical uncertainty for any measurement of an isotope \( E \), such that

\[u_{E_{\text{streamflow}}} = u_{E_{\text{rain}}} = u_{E_{\text{baseflow}}} = u_E\]

With this approach, I get \( p_{\delta^{18} O} = 0.99685 \pm 0.1594934 \) and \( p_{\delta^2 H} = 1.0259682 \pm 0.1413952 \), such that following Equation 2.2 \( p = 1.0114091 \pm 0.1065724 \).

References